
- Lecture 1: Theory of copula entropy (1)
- Lecture 2: Theory of copula entropy (2)
- Lecture 3: Theoretical applications (1)
- Lecture 4: Theoretical applications (2)
- Lecture 5: Theoretical applications (3)
- Lecture 6: Evaluation
- Lecture 7: Generalizations
- **Lecture 8: Real applications and softwares**

Lecture 8: Real applications and softwares

1 Real applications

- Astronomy
- Physics
- Chemistry
- Geology
- Biology
- Medicine
- Social sciences
- Engineering

2 Softwares

- Official
- Third-party

- 测光红移
 - 测光红移准确度提升¹

- 相关粒子系统
 - 平衡态相关粒子系统中熵的推导和计算²

²Jian Ma. "On Thermodynamic Interpretation of Copula Entropy". In: *arXiv preprint arXiv:2111.14042* (2021). arXiv: 2111.14042 [cs.IT].

- 赤道电离层特性随季节变化分析
 - 基于 TE 分析赤道地区高频微波信号传输中 TEC 和 DFS 之间的非线性关系³

³ Aderonke Akerele et al. "Complexity and Nonlinear Dependence of Ionospheric Electron Content and Doppler Frequency Shifts in Propagating HF Radio Signals within Equatorial Regions". In: *Atmosphere* 15.6 (2024), p. 654. DOI: 10.3390/atmos15060654. URL: <https://www.mdpi.com/2073-4433/15/6/654>.

- *Journal of the American Medical Association*, 1994

- 分子设计
 - 设计具有特定属性的分子结构⁹

⁹Mario Wieser. “Learning Invariant Representations for Deep Latent Variable Models”. PhD thesis. University of Basel, 2020.

¹¹ Song Zhang et al. "Correlation between plastic deformation mechanism and texture evolution of a near β -Ti alloy deformed in β region". In: *Intermetallics* 170 (2024), p. 108333. ISSN: 0966-9795. DOI: <https://doi.org/10.1016/j.intermet.2024.108333>. URL: <https://www.sciencedirect.com/science/article/pii/S0966979524001523>.

CONCLUSIONS

- 土地干旱度分类
 - 考虑蒸散过程的土地干旱度分类¹⁸
- 北欧山地陆气耦合分析
 - 利用 CE 分析了芬瑟两个观测站的气象观测变量之间 CE 和解耦度量之间的关系¹⁹

¹⁹Laura Mack, Marvin Kähnert, and Norbert Pirk. “Probabilistic modelling of atmosphere-surface coupling with a copula Bayesian network”. In: *Environmental Modelling & Software* (2026). arXiv: 2509.11975 [physics.ao-ph]. URL: <https://arxiv.org/abs/2509.11975>.

水文学 1

- 洪水预报
 - 金沙江流域洪水预报²⁰
 - 天山北麓奎屯河洪水分析²¹
- 河流相关性
 - 长江上游河段（金沙江、岷江、沱江、嘉陵江）相关性²²
 - 长江上游河段复合洪水事件分析²³
- 水沙关系分析
 - 黄河西柳沟河流域径流量和输沙量数据分析²⁴

²⁰ Lu Chen, Vijay P. Singh, and Shenglian Guo. "Measure of Correlation between River Flows Using the Copula-Entropy Method". In: *Journal of Hydrologic Engineering* 18.12 (2013), pp. 1591-1606. Lu Chen et al. "Copula entropy coupled with artificial neural network for rainfall-runoff simulation". In: *Stochastic Environmental Research and Risk Assessment* 28.7 (2014), pp. 1755-1767. 陈佳雷 et al. "一种基于多要素注意力时空图卷积网络的经济预报方法". Pat. CN117151285A, CN117151285A, 2023.

²¹Yingying Han et al. "Non-Stationary Flood Characteristics and Joint Risk Analysis in Inland China with Uncertainty Considerations". In: *Atmosphere* 17,3 (2026). p. 281.

²²Lu Chen and Shenglian Guo. *Copulas and its Application in Hydrology and Water Resources*. Springer Singapore, 2018.

²³Xu Wang and Yong-Ming Shen. "A Framework of Dependence Modeling and Evaluation System for Compound Flood Events". In: *Water Resources Research* 59.8 (2023), e2023WR034718. DOI: <https://doi.org/10.1029/2023WR034718>.

²⁴ Longxia Qian et al. "A New Estimation Method for Copula Parameters for Multivariate Hydrological Frequency Analysis With Small Sample Sizes". In: *Water Resources Management* 36.4 (2022), pp. 1141–1157. ISSN: 1573-1650. URL: <https://doi.org/10.1007/s11269-021-03016-w>.

- 干旱研究

- 黄河流域干旱分析和预测²⁵
- 黄河流域干旱事件识别²⁶
- 印度达布蒂 (Tapti) 河流域干旱指数设计²⁷
- 伊朗城市 (扎黑丹、恩泽利和马什哈德) 干旱数据分析²⁸
- 美国科罗拉多河流域干旱传播分析²⁹
- 黑龙江讷谟尔河流域干旱形成和演化分析³⁰
- 巴基斯坦旁遮普省干旱分析³¹

³¹ Muhammad Owais Khan et al. "Assessing drought dynamics across climatic zones using parametric and entropy copula models". In: *Stochastic Environmental Research and Risk Assessment* 40.8 (July 2026), p. 191. ISSN: 1436-3259. URL: <https://doi.org/10.1007/s00477-026-03313-z>.

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- 气候变化对水文气象变量相关性的影响
 - 美国德州达拉斯地区降水和气温相关性关系分析⁴⁴
- 气候评估
 - 欧洲城市气候分类⁴⁵

⁴⁴ Zengchao Hao and Vijay P. Singh. "Integrating Entropy and Copula Theories for Hydrologic Modeling and Analysis". In: *Entropy* 17.4 (2015), pp. 2253–2280. ISSN: 1099-4300. DOI: 10.3390/e17042253. URL: <https://www.mdpi.com/1099-4300/17/4/2253>.

⁴⁵ **Francesca Condino**. "La divergenza di Jensen-Shannon nell'algoritmo di clustering dinamico per dati descritti da distribuzioni multivariate". PhD thesis. Università degli Studi di Napoli Federico II, 2009.

气象学

- 大气污染气象成因分析
 - 北京地区气象因素对 PM2.5 浓度的因果关系分析⁴⁶
 - PM2.5 浓度预测⁴⁷
 - 上海和广州大气污染预测预警⁴⁸
- 气象灾害预测
 - 广西地区台风灾情预测类⁴⁹

⁴⁶Jian Ma. "Estimating Transfer Entropy via Copula Entropy". In: *arXiv preprint arXiv:1910.04375* (2019).

⁴⁷ Xiaoxuan Wu et al. "Research on PM2.5 Concentration Prediction Based on the CE-AGA-LSTM Model". In: *Applied Sciences* 12.14 (2022), p. 7009. ISSN: 2076-3417. DOI: 10.3390/app12147009. URL: <https://www.mdpi.com/2076-3417/12/14/7009>. Jieyin Chen.

"Short-Term Prediction of PM2.5 Concentration based on Self-Attention Mechanism Improved Temporal Convolution Network". In: 2023 International Seminar on Computer Science and Engineering Technology (SCSET). 2023, pp. 528–534. DOI: 10.1109/SCSET58950.2023.00121.

⁴⁸ **Jujie Wang et al.** "A novel air quality prediction and early warning system based on combined model of optimal feature extraction and intelligent optimization". In: *Chaos, Solitons & Fractals* 158 (2022), p. 112098. ISSN: 0960-0779. DOI: <https://doi.org/10.1016/j.chaos.2022.112098>. URL: <https://www.sciencedirect.com/science/article/pii/S0960077922003083>.

⁴⁹ 陈燕游, 刘合香, and 倪增华. “基于 Copula 熵因子选取的 PSO-ELM 台风灾情预测模型”. In: 气象研究与应用 40.2 (2019), pp. 7–11.

生态学

- 动物运动轨迹分析
 - Cylcop 算法包⁵⁰
- 土壤生态学
 - 利用微生物-酶分解模型研究了土壤湿度变化对土壤碳氮动态性的长期影响⁵¹
- 社会生态学
 - 利用基于 CE 的 TE 和网络分析等工具研究西南喀斯特地区社会生态系统脆弱性演化⁵²

⁵⁰Florian H. Hodel and John R. Fieberg. "cylcop: An R Package for Circular-Linear Copulae with Angular Symmetry". In: *bioRxiv* (2021), p. 2021.07.14.452253, Florian Hodel. *cylcop: Circular-Linear Copulas with Angular Symmetry for Movement Data*. CRAN. R package version 0.2.0, URL: <https://cran.r-project.org/package=cylcop>. 2022.

⁵¹Wanyu Li et al. "Microbially-Mediated Soil Carbon-Nitrogen Dynamics in Response to Future Soil Moisture Change". In: *Earth's Future* 13.3 (2025), e2024EF005521. DOI: <https://doi.org/10.1029/2024EF005521>.

⁵²Yueying Li et al. "Integrating Transfer Entropy and Network Analysis to Explore Social-Ecological Resilience Evolution: a case study in South China Karst". In: *Journal of Cleaner Production* (2025), p. 145926. ISSN: 0959-6526. DOI: <https://doi.org/10.1016/j.jclepro.2025.145926>. URL: <https://www.sciencedirect.com/science/article/pii/S0959652625012764>.

⁵⁴Soumik Purkayastha and Peter X.K. Song. "Asymmetric predictability in causal discovery: an information theoretic approach". In: *arXiv preprint arXiv:2210.14455* (2022).

- 气候变化对我国南方两季稻产量的影响及对策⁵⁶
- 南方水稻产量预测⁵⁷
- 南极磷虾分布预测⁵⁸

⁵⁶ Ziya Zhang et al. "Impact of climate change and planting date shifts on growth and yields of double cropping rice in southeastern China in future". In: *Agricultural Systems* 205 (2023), p. 103581. ISSN: 0308-521X. DOI: <https://doi.org/10.1016/j.agry.2022.103581>. URL: <https://www.sciencedirect.com/science/article/pii/S0308521X22002177>, 张子雅. "气候变化对中国水稻物候和产量的影响及播期优化". 硕士学位论文. 西北农林科技大学, 2023.

⁵⁷张春磊 et al. “基于深度学习技术的水稻环境因素产量预测”. In: 电子技术应用 50.4 (2024), pp. 81–86.

⁵⁸王嘉龙, “南极磷虾及其相关种空间分布对未来气候变化的响应研究”. 硕士学位论文, 上海海洋大学, 2024.

免疫学

- 淋巴细胞间通信机制研究
 - 提出利用 TE 度量来分析小鼠 B 细胞活体钙离子成像⁵⁹

⁵⁹ 安部 武志, 浅井 義之, and 安達 貴弘. “免疫細胞カルシウム濃度動態に基づく未病^⑤出のための高次グラフシグナル解析”. In: 生体医学 Annual63.Proc (2025). pp. 430–432. DOI: 10.11239/jsmbe.Annual63.430.

神经科学 1

- 神经病学

- 注意缺陷多动障碍⁶⁰
- 癫痫症⁶¹
- 脑卒中⁶²

⁶⁰ Paolo Victor Redondo, Raphaël Huser, and Hernando Ombao. "Measuring Information Transfer Between Nodes in a Brain Network through Spectral Transfer Entropy". In: *arXiv preprint arXiv:2303.06384* (2023). 汪方毅 et al. "基于静息态 fMRI 区分健康老年人认知水平的 MVPA 方法研究". In: *磁共振成像* 14.6 (2023), pp. 18–25.

⁶¹Zhaohui Li et al. "Estimating global phase synchronization by quantifying multivariate mutual information and detecting network structure". In: *Neural Networks* 183 (2025), p. 106984. ISSN: 0893-6080. DOI: <https://doi.org/10.1016/j.neunet.2024.106984>. URL: <https://www.sciencedirect.com/science/article/pii/S0893608024009134>.

⁶²Wojciech Ciezobka et al. "End-to-End Stroke Imaging Analysis Using Effective Connectivity and Interpretable Artificial Intelligence". In: *IEEE Access* 13 (2025), pp. 10227–10239. DOI: 10.1109/ACCESS.2025.3529179.

- 认知神经学

- 分析大脑认知活动的多模态数据⁶³
- 语音信息的编码和解析⁶⁴
- 神经元特异化识别⁶⁵

⁶³Stephanie J. Kayser et al. "Irregular Speech Rate Dissociates Auditory Cortical Entrainment, Evoked Responses, and Frontal Alpha". In: *The Journal of Neuroscience* 35.44 (2015), pp. 14691–14701, Robin A. A. Ince et al. "The Deceptively Simple N170 Reflects Network Information Processing Mechanisms Involving Visual Feature Coding and Transfer Across Hemispheres". In: *Cerebral Cortex* 26.11 (2016), pp. 4123–4135, Robin A.A. Ince et al. "A statistical framework for neuroimaging data analysis based on mutual information estimated via a gaussian copula". In: *Human Brain Mapping* 38.3 (2017), pp. 1541–1573, Etienne Combrisson et al. "Group-level inference of information-based measures for the analyses of cognitive brain networks from neurophysiological data". In: *NeuroImage* 258 (2022), p. 119347. ISSN: 1053-8119. DOI: <https://doi.org/10.1016/j.neuroimage.2022.119347>. URL: <https://www.sciencedirect.com/science/article/pii/S1053811922004669>.

⁶⁴Pieter De Clercq et al. "Beyond linear neural envelope tracking: a mutual information approach". In: *Journal of Neural Engineering* 20.2 (2023), p. 026007. DOI: 10.1088/1741-2552/acbe1d. URL: <https://dx.doi.org/10.1088/1741-2552/acbe1d>.

⁶⁵ NA Pospelov et al. "Searching for cognitive specializations of neurons using mutual information framework". In: *Genes & Cells* 18.4 (2023), pp. 878–881.

神经科学 3

- 运动神经学
 - 分析运动的肌肉组合协同策略⁶⁶
- 计算神经学
 - 神经元可塑性建模⁶⁷
 - 神经网络信息传输关系分析⁶⁸

⁶⁶ 吴亚婷 et al. “多尺度肌间耦合网络分析”. In: 生物医学工程学杂志 38.4 (2021), pp. 742–752, Yating Wu et al. “R-Vine Copula Mutual Information for Intermuscular Coupling Analysis”. In: *Proceedings of the 11th International Conference on Computer Engineering and Networks*. 2022, pp. 526–534, David O’ Reilly and Ioannis Delis. “A network information theoretic framework to characterise muscle synergies in space and time”. In: *Journal of Neural Engineering* 19.1 (2022), p. 016031. DOI: 10.1088/1741-2552/ac5150. URL: <https://doi.org/10.1088/1741-2552/ac5150>, Shaojun Zhu et al. “Intermuscular coupling network analysis of upper limbs based on R-vine copula transfer entropy”. In: *Mathematical Biosciences and Engineering* 19.9 (2022), pp. 9437–9456, 金国美 et al. “基于小波包-Copula 互信息的肌间耦合特性”. In: 传感技术学报 35.10 (2022), pp. 1348–1353.

⁶⁷ Johannes Leugering and Gordon Pipa. “A Unifying Framework of Synaptic and Intrinsic Plasticity in Neural Populations.”. In: *Neural Computation* 30.4 (2018), pp. 945–986.

⁶⁸ Ari Pakman et al. “Estimating the Unique Information of Continuous Variables in Recurrent Networks”. In: *Advances in Neural Information Processing Systems* 34 (2021), pp. 20295–20307.

心理学

- 生物心理学
 - 情绪刺激下心跳诱发脑电位的时间交互现象⁶⁹

⁶⁹ Liesa Ravijs. "Revealing temporal interactions around the heartbeat-evoked potential modulated by emotional perception". MA thesis. Ghent University, 2019.

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● 临床医学 2

- 白内障术后角膜水肿风险预测⁷⁸
- 主动脉瓣置换手术射血分数分析⁷⁹
- 脑肿瘤医学影像组学诊断模型构建⁸⁰
- 基于脉搏波的高血压和糖尿病健康状态分类⁸¹

⁷⁸ Yu Luo et al. "Research on Establishing Corneal Edema after Phacoemulsification Prediction Model Based on Variable Selection with Copula Entropy". In: *Journal of Clinical Medicine* 12.4 (2023), p. 1290. ISSN: 2077-0383. DOI: 10.3390/jcm12041290. URL: <https://www.mdpi.com/2077-0383/12/4/1290>.

⁷⁹S.M. Sunoj and N. Unnikrishnan Nair. "Survival Copula Entropy and Dependence in Bivariate Distributions". In: *REVSTAT-Statistical Journal* 23.1 (2025), pp. 101–115. URL: <https://revstat.ine.pt/index.php/REVSTAT/article/view/560>.

⁸⁰ 潘红宇, “基于影像组学与深度学习的脑肿瘤图像分类研究”. 硕士学位论文. 西南大学, 2023.

⁸¹ 汤宇飞, “基于脉搏波的糖尿病和高血压诊断算法研究”, 硕士学位论文, 中国矿业大学, 2023.

医学 3

- 认知医学
 - 认知能力评估 / 痴呆症筛查⁸²
- 运动医学
 - 运动能力评估 / 跌倒风险预测⁸³
 - 重复经颅磁刺激对帕金森病改善神经机制分析⁸⁴
- 精神病学
 - 抑郁症患者识别⁸⁵

⁸² Jian Ma. "Predicting MMSE Score from Finger-Tapping Measurement". In: *Proceedings of 2021 Chinese Intelligent Automation Conference*. See also bioRxiv 817338 (2019). 2022, pp. 294–304. ISBN: 978-981-16-6372-7.

⁸³ Jian Ma. "Predicting TUG score from gait characteristics based on video analysis and machine learning". In: *Proceedings of 2023 Chinese Intelligent Automation Conference*. See also bioRxiv 963686 (2020). 2023, pp. 1–12, Jian Ma. "Associations between finger tapping, gait and fall risk with application to fall risk assessment". In: *arXiv preprint arXiv:2006.16648* (2020).

⁸⁴ 李润泽 et al. "重复经颅磁刺激改善帕金森病运动症状的脑功能网络分析". In: *生物化学与生物物理进展* 50.1 (2023), pp. 126–134.

⁸⁵ 张婷婷 et al. "基于 Couple 熵的抑郁症相干性反馈指标提取". In: *电子测量技术* 45.9 (2022), pp. 160–167.

医学 4

- 法医学
 - 基于 DNA 分析推断人身高⁸⁶

⁸⁶王中华. “人类身高的表观基因组关联分析和基于 DNA 甲基化构建成人身高推断的深度神经网络模型”. 博士学位论文. 河北医科大学, 2024.

- 药物-靶点相互作用预测
 - 提出了一种交互式多特征融合的 DTIs 预测算法⁸⁷

⁸⁷ 高浩田 et al. “基于交互式多特征融合算法的药物靶标预测”. In: 太原理工大学学报 55.4 (2024), pp. 751–758.

- 新冠肺炎流行病 (COVID19)
 - 发热症状疑似病人筛查诊断⁸⁸
- 高血压
 - 高血压关联基因研究⁸⁹

⁸⁹ Soumik Purkayastha and Peter X.K. Song. "Asymmetric predictability in causal discovery: an information theoretic approach". In: *arXiv preprint arXiv:2210.14455* (2022).

● 经济学

- 扶贫政策效果评估, 用于政策目标人口鉴别⁹⁰
- 议价机制中的互惠行为和时间效应⁹¹
- 产业链内部相关性分析⁹²
- 投资者情绪分析⁹³
- 通胀与预期关系分析⁹⁴

⁹¹ [Leonie Bossemeyer](#). "Machine Learning for Causal Discovery with Applications in Economics". MA thesis. Ludwig-Maximilians-Universität München. 2021.

⁹² 韦颖璐, “基于 pair-copula 熵的相关性度量”, 硕士学位论文, 苏州大学, 2021.

⁹³ Muye Han and Jinsheng Zhou. "Multi-Scale Characteristics of Investor Sentiment Transmission Based on Wavelet, Transfer Entropy and Network Analysis". In: *Entropy* 24.12 (2022), p. 1786. ISSN: 1099-4300. DOI: 10.3390/e24121786. URL: <https://www.mdpi.com/1099-4300/24/12/1786>.

⁹⁴ Omid M. Ardakani. "Information Content of Inflation Expectations: A Copula-Based Model". In: *Studies in Nonlinear Dynamics & Econometrics* 29.1 (2024), pp. 71–93. DOI: doi:10.1515/snd-2023-0075. URL: <https://doi.org/10.1515/snd-2023-0075>.

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¹⁰³ Bowen Zhang et al. "Changes in Public Sentiment under the Background of Major Emergencies – Taking the Shanghai Epidemic as an Example". In: *International Journal of Environmental Research and Public Health* 19.19 (2022), p. 12594. ISSN: 1660-4601. DOI: 10.3390/ijerph191912594. URL: <https://www.mdpi.com/1660-4601/19/19/12594>.

社会科学 4

- 法学
 - 社区属性与社区犯罪关系分析¹⁰⁴
- 政治学
 - 分析政权领导力因素和政权危机之间关系¹⁰⁵
- 军事学
 - 目标意图识别¹⁰⁶
- 情报学
 - 颠覆性技术科学-技术-产业互动模式分析¹⁰⁷
 - 高价值专利评估和发现¹⁰⁸

¹⁰⁴ Mario Wieser. "Learning Invariant Representations for Deep Latent Variable Models". PhD thesis. University of Basel, 2020.

¹⁰⁵ Stuart William Card. "Towards an Information Theoretic Framework for Evolutionary Learning". PhD thesis. Syracuse University, 2011.

¹⁰⁶ 张可 et al. "一种基于动态贝叶斯网络的目标意图识别方法". Pat. CN114997306A. CN114997306A. 2022.

¹⁰⁷ 许海云 et al. "颠覆性技术的科学-技术-产业互动模式识别与分析". In: 情报学报 42.7 (2023), pp. 816–831.

¹⁰⁸ 徐蕊婷, 陈哲宇, and 陈加鹏. "基于两阶段混合式特征选择的潜在高价值专利多情景判别方法研究". In: 数据分析与知识发现 (2026).

工学 1

● 能源工程 1

- 能源网络管理，研究天气因素与能源网络的耦合¹⁰⁹
- 光伏发电功率预测¹¹⁰
- 风电机组工况划分¹¹¹
- 电力负荷预测¹¹²
- 风光储协同规划¹¹³

¹⁰⁹ Xueqian Fu et al. "Uncertainty analysis of an integrated energy system based on information theory". In: *Energy* 122.122 (2017), pp. 649–662.

¹¹⁰ 朱正林 和 张冕. "基于 AO 优化 VMD-CE-BiGRU 的光伏发电功率预测". In: *国外电子测量技术* 41.10 (2022), pp. 56–61.

¹¹¹ 崔双双 和 孙单勋. "分工况下风电机组各变量相关性研究". In: *综合智慧能源* 44.12 (2022), pp. 49–55.

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工学 2

● 能源工程 2

- 电网频率稳定性预测¹¹⁴
- 用户线损贡献分析¹¹⁵
- 配电网拓扑辨识¹¹⁶
- 电价预测¹¹⁷
- 电力系统宽频振荡影响因素和传播路径分析¹¹⁸
- 电力系统信息物理融合¹¹⁹

¹¹⁴Peili Liu et al. "Frequency Stability Prediction of Power Systems Using Vision Transformer and Copula Entropy". In: *Entropy* 24.8 (2022), p. 1165. ISSN: 1099-4300. DOI: 10.3390/e24081165. URL: <https://www.mdpi.com/1099-4300/24/8/1165>.

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工学 3

- 能源工程 3
 - 锂电池容量估计¹²⁰
 - 能效异常原因诊断¹²¹

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工学 4

- 纺织工程
 - 纱布生产过程质量控制¹²²
 - 可穿戴织物传感器方案设计¹²³
- 食品工程
 - 葡萄酒质量与理化成分关系分析¹²⁴

¹²²Seyedeh Azadeh Fallah Mortezaejad et al. "Non-Parametric Multivariate Control Chart Using Copula Entropy". In: *Sankhya B* 87 (2025), pp. 468–518. URL: <https://doi.org/10.1007/s13571-025-00374-y>.

¹²³Jiaqi Mo et al. "An Evaluation Framework for Textile Stretch Sensor Layouts Based on Mutual Information". In: *Companion of the 2025 ACM International Joint Conference on Pervasive and Ubiquitous Computing*. UbiComp Companion '25. Finland: Association for Computing Machinery, 2026, pp. 35–39. ISBN: 9798400714771. DOI: 10.1145/3714394.3754434. URL: <https://doi.org/10.1145/3714394.3754434>.

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工学 5

● 土木建筑

- 建筑能源系统节能技术¹²⁵
- 工程变形监测¹²⁶
- 钢筋混凝土梁地震性能等级预测¹²⁷
- 受剪承载力预测¹²⁸
- 医院建筑系统脆弱性评估¹²⁹

¹²⁵ Zhiwei Li et al. "A model-free method for identifying time-delay characteristics of HVAC system based on multivariate transfer entropy". In: *Building and Environment* 217 (2022), p. 109072. ISSN: 0360-1323. DOI:

<https://doi.org/10.1016/j.buildenv.2022.109072>. URL: <https://www.sciencedirect.com/science/article/pii/S0360132322003110>.

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¹²⁷ Chao Ma et al. "Prediction on seismic performance levels of reinforced concrete beams by considering crack development". In: *Soil Dynamics and Earthquake Engineering* 187 (2024), p. 109006, 迟经纬. "基于承载与防水功能的城市地下空间结构抗震韧性评价方法". 硕士学位论文. 北京建筑大学, 2024.

¹²⁸ 常旭 et al. "RC 柱受剪承载力神经网络预测模型及其可解释性". In: *新疆大学学报 (自然科学版中英文)* 42.1 (2025), pp. 114–128.

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工学 6

● 交通运输

- 大件货物运输方案制定¹³⁰
- 航空和高铁票价影响因素分析¹³¹
- 城市轨道交通客流分析和预测¹³²
- 铁路客流量预测¹³³

¹³⁰ 黄达. “基于区块链构建的大件货物多式联运方案研究”. 博士学位论文. 北京交通大学, 2021.

¹³¹ 许罗豪 et al. “基于熵与回归树的票价影响因素研究”. In: *综合运输* 45.6 (2023), pp. 125–130.

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¹³³ Jiahao Chang and Xiaoyu Song. “A Railway Passenger Flow Prediction Model Based on Improved Prophet”. In: *Proceedings of the 2023 4th International Conference on Machine Learning and Computer Application*. 2024, pp. 798–804. doi: 10.1145/3650215.3650354, 常家豪. “基于集成模型的铁路客流量预测研究”. 硕士学位论文. 兰州交通大学, 2024.

- 制造工程

- 制造质量管理，研究优化制造过程参数，预测产品质量¹³⁴
- 装配质量控制¹³⁵
- 工业过程故障监测¹³⁶
- 钢铁工艺过程碳排放预测¹³⁷
- 液晶显示器质量预测¹³⁸
- 航空发动机涡轮盘制造工艺优化¹³⁹

¹³⁴ Yan-Ning Sun et al. "Modelling and Prediction of Injection Molding Process Using Copula Entropy and Multi-Output SVR". In: *IEEE 17th International Conference on Automation Science and Engineering*. 2021, pp. 1677–1682.

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¹³⁷ Jie Hu et al. "Dynamic Modeling Framework Based on Automatic Identification of Operating Conditions for Sintering Carbon Consumption Prediction". In: *IEEE Transactions on Industrial Electronics* 71.3 (2023), pp. 3133–3141. DOI: 10.1109/TIE.2023.3270514.

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工学 8

● 可靠性工程

- 系统退化过程建模¹⁴⁰
- 风电机组健康状态评估¹⁴¹
- 砂轮剩余寿命预测¹⁴²
- 滚动轴承剩余使用寿命预测¹⁴³

¹⁴⁰ Fuqiang Sun et al. "A Copula Entropy Approach to Dependence Measurement for Multiple Degradation Processes". In: *Entropy* 21.8 (2019), p. 724.

¹⁴¹ 齐咏生 et al. "一种基于多维度 SCADA 数据评估风电机组健康状态评估方法". Pat. CN110442833A. CN110442833A. 2019.

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工学 9

- 石油工程
 - 煤层气井产量预测¹⁴⁴
 - 陆地原油生产碳排放分析¹⁴⁵
- 矿业工程
 - 煤-岩识别¹⁴⁶
 - 重介质选煤¹⁴⁷

¹⁴⁴ **Zhifeng Luo and Haojiang Xi.** "Hybrid Model Based on Copula Mutual Information and SSA-BP: Analysis of Key Factors and Prediction of Stable Gas Production". In: *Arabian Journal for Science and Engineering* 50 (2025), pp. 4673–4685. ISSN: 2191-4281. URL: <https://doi.org/10.1007/s13369-024-09205-0>.

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工学 10

- 医学工程
 - 脑机接口¹⁴⁸
- 冶金工程
 - 真空蒸馏法制备高纯金属工艺参数优化¹⁴⁹

¹⁴⁸Chao Tang, Dongyao Jiang, and Badong Chen. "MEG Channel Selection Using Copula Entropy-Based Transfer Entropy for Motor Imagery BCI". In: *2024 46th Annual International Conference of the IEEE Engineering in Medicine and Biology Society (EMBC)*. 2024, pp. 1–4. DOI: 10.1109/EMBC53108.2024.10782066, Chao Tang et al. "Copula Transfer Entropy-Based Channel Selection for MEG Motor Imagery Brain Computer Interfaces". In: *Tsinghua Science and Technology* 31.3 (2026), pp. 1474–1486. DOI: 10.26599/TST.2025.9010006. URL: <https://www.sciopen.com/article/10.26599/TST.2025.9010006>.

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工学 11

● 化学工程

- 化学过程故障监测和诊断¹⁵⁰
- 化工过程因果网络构建¹⁵¹
- 化工过程缺失数据补全¹⁵²
- 生物发酵过程软测量¹⁵³

¹⁵⁰Min Yin, Jince Li, and Hongguang Li. "A CNN approach based on correlation metrics to chemical process fault classifications with limited labeled data". In: *The Canadian Journal of Chemical Engineering* 101.7 (2022), pp. 3982–3997. doi: 10.1002/cjce.24749. eprint: <https://onlinelibrary.wiley.com/doi/pdf/10.1002/cjce.24749>. URL: <https://onlinelibrary.wiley.com/doi/abs/10.1002/cjce.24749>, Yingpeng Wei and Li Wang. "Copula entropy-based PCA method and application in process monitoring". In: *2022 4th International Conference on Intelligent Information Processing (IIP)*. 2022, pp. 61–64. DOI: 10.1109/IIP57348.2022.00019, Shuangshuang Pan, Li Zhu, and Xirong Xu. "Root cause and fault propagation analysis based on causal graph in chemical processes". In: *2023 CAA Symposium on Fault Detection, Supervision and Safety for Technical Processes (SAFEPROCESS)*. 2023, pp. 1–6. DOI: 10.1109/SAFEPROCESS58597.2023.10295717.

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工学 12

- 核工程
 - 核反应堆传感器故障与事故预防¹⁵⁴

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- 航空航天

- 飞行器总体参数分析和优化¹⁵⁵
- 卫星在轨健康状态监测¹⁵⁶
- 涡扇发动机健康状态监测¹⁵⁷
- 机场间航班延误因果关系分析¹⁵⁸
- 航班到达时间预测¹⁵⁹

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工学 14

- 兵器工程
 - 武器效能评估指标体系约简¹⁶⁰
- 车辆工程
 - CAN 总线入侵检测¹⁶¹
 - 高速动车组关键部件轴承健康监测¹⁶²
 - 自动驾驶感知模块优化¹⁶³
- 控制工程
 - 航站楼室内温度控制¹⁶⁴

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工学 15

- 电子工程
 - 集成电路封装材料物理性能预测¹⁶⁵
- 通信工程
 - 通讯网络加密技术研究¹⁶⁶
 - 6G 网络语义通信技术研究¹⁶⁷
 - 游戏系统编解码技术¹⁶⁸
- 高性能计算
 - 高性能计算能源效率优化¹⁶⁹
- 信息安全
 - 深度神经网络对抗攻击防御算法¹⁷⁰

¹⁶⁵ 刘勃, “基于机器学习的封装材料加速预测”, 硕士学位论文, 哈尔滨理工大学, 2022.

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工学 16

- 测绘遥感
 - 高光谱遥感数据分析¹⁷¹
 - 大型工程变形体内部监测¹⁷²
- 海洋工程
 - 海洋底质信息探测¹⁷³

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金融工程 1

- 投资组合优化
 - 股票资产相关性网络分析¹⁷⁴
 - ST 股票分类¹⁷⁵
- 金融问题建模
 - Copula 函数模型选择¹⁷⁶
- 股票相关性建模
 - R-vine copula 结构建模¹⁷⁷

¹⁷⁴ Qitong Wang. "Social Networks, Asset Allocation and Portfolio Diversification". MA thesis. University of Waterloo, 2015, 廖轶楠. "基于 Copula 熵选股及集成神经网络预测的投资组合管理研究". 硕士学位论文. 南京信息工程大学, 2023.

¹⁷⁵ 朱仲儿. "多种机器学习方法的股票分类预测". 硕士学位论文. 上海师范大学, 2022.

¹⁷⁶ Rafael Calsaverini and Renato Vicente. "An information-theoretic approach to statistical dependence: Copula information". In: *EPL (Europhysics Letters)* 88.6 (2009), p. 68003. DOI: 10.1209/0295-5075/88/68003. URL: <https://doi.org/10.1209/0295-5075/88/68003>.

¹⁷⁷ Fadiah Amer Alanazi. "Truncating Regular Vine Copula Based on Mutual Information: An Efficient Parsimonious Model for High-Dimensional Data". In: *Mathematical Problems in Engineering* 2021 (2021), p. 4347957. URL: <https://doi.org/10.1155/2021/4347957>, 王念鸽. "基于互信息的 Vine Copula 模型的高频数据投资组合风险测度研究". 硕士学位论文. 浙江财经大学, 2023.

金融工程 2

- 量化金融工具箱 MLFinLab¹⁷⁸
- 金融系统性风险
 - 行业风险溢出效应分析¹⁷⁹
 - 金融脆弱性度量¹⁸⁰
- 信用风险评价
 - 信用风险卡模型建立¹⁸¹
 - P2P 借贷个人信用风险模型¹⁸²

¹⁷⁸Hudson and Thames. *Machine Learning Financial Laboratory (MLFinLab)*. GitHub. 2021. URL: <https://github.com/hudson-and-thames/mlfinlab>.

¹⁷⁹熊靖宇. “基于 Copula 熵的行业风险溢出效应分析”. 硕士学位论文. 东北财经大学, 2020.

¹⁸⁰Wanfang Chen and Marc G Genton. “Are you all normal? It depends!” In: *International Statistical Review* 91.1 (2023), pp. 114–139.

¹⁸¹孔祥永 et al. “一种自动化特征工程信用风险评价系统及方法”. Pat. CN114049198A. CN114049198A. 2021.

¹⁸²彭翊庭. “个人信用风险评估模型比较——基于 Copula 熵的特征选择”. 硕士学位论文. 清华大学, 2022.



金融工程 3

- 金融产品价格预测
 - 基于因果关系的迁移学习价格预测模型构建¹⁸³
- 疫情对金融市场的影响
 - 疫情对股票指数相关性的影响¹⁸⁴
- 金融市场预测
 - 加密货币市场预测¹⁸⁵

¹⁸³Dabin Zhang et al. "A novel deep transfer learning framework with adversarial domain adaptation: application to financial time-series forecasting". In: *Neural Computing and Applications* 35 (2023), pp. 24037–24054. URL: <https://doi.org/10.1007/s00521-023-09047-1>.

¹⁸⁴Henryk Gurgul and Robert Syrek. "Mutual information between Polish subindexes –the use of copula entropy around the time of the COVID-19 pandemic". In: *Statistics in Transition new series* 25.1 (2024), pp. 23–41. DOI: 10.59170/stattrans-2024-002.

¹⁸⁵Amanda Mahmutovic. "Forecasting Cryptocurrency Returns In Adaptive Markets With an Extended Copula Based Feature Selection and Extended Copula Divergence Hybrid Loss Function". MA thesis. Linnaeus University, 2025.

- **copent**¹⁸⁶

The **copent** package in R and Python for estimating copula entropy, transfer entropy, the statistic for multivariate normality test, two-sample test, and change point detection are available on CRAN and PyPI respectively. The source codes are provided on GitHub.



<https://cran.r-project.org/package=copent>



<https://pypi.org/project/copent/>



<https://github.com/majianthu>

¹⁸⁶ [Jian Ma](#). "copent: Estimating Copula Entropy and Transfer Entropy in R". In: *arXiv preprint arXiv:2005.14025* (2021).

¹⁸⁹Hudson and Thames. *The ArbitrageLab package in Python*. GitHub. 2024. URL: <https://github.com/hudson-and-thames/mlfinlab>.

- Python: **gcml**¹⁹⁰ (by Glasgow University, Radboud University Nijmegen, University of Amsterdam, Kyiv AI group, Github 100+ stars), **pytorch-mighty**¹⁹¹ (by Kyiv AI group), **HOI**¹⁹² and **Frites**¹⁹³ (by CNRS, INT, and Aix-Marseille Université), **Tensorpac**¹⁹⁴ (by Université de Montréal, INT, and Aix-Marseille Université), **driada**¹⁹⁵ (by Moscow State University);

¹⁹⁵ [Institute for Advanced Brain Studies. *driada*: Dimensionality Reduction for Integrated Activity Data. Github. URL: https://github.com/iabs-neuro/driada. 2024.](https://github.com/iabs-neuro/driada)

Third-party

- **Python:** `CopulaGP`¹⁹⁶ (by University of Edinburgh), `Polars-ds`¹⁹⁷ (by New York University, 1.45+ million downloads, Github 650+ stars), `copul`¹⁹⁸ (by Albert-Ludwigs-Universität Freiburg), `rcopula`¹⁹⁹, and `pycopcor`²⁰⁰ (by Technische Universität Dresden);

²⁰⁰Andreas Gocht-Zech. *The pycopcor package in Python*. Github. URL: <https://github.com/AndreasGocht/pycopcor>. 2023.

Softwares

Third-party

- Julia: `CopEnt.jl`²⁰¹ (by University of Edinburgh), `CausalityTools.jl`²⁰² (by JuliaDynamics), and `Copulas.jl`²⁰³ (by Aix-Marseille Université, Github 110+ stars);
- Matlab: `gcmi`²⁰⁴ (by Glasgow University, Radboud University Nijmegen, University of Amsterdam, Kyiv AI group, Github 100+ stars), and `FieldTrip`²⁰⁵ (by Radboud University Nijmegen);
- C++: `NPStat`^{206 207} (by Texas Tech University).

²⁰¹Kai Xu. *The CopEnt.jl package in Julia*. Github. 2021. URL: <https://github.com/xukai92/CopEnt.jl>.

²⁰²Kristian Agasøster Haaga. *The CausalityTools.jl package v2.10.1 in Julia*. Github. 2023. URL: <https://github.com/JuliaDynamics/CausalityTools.jl>.

²⁰³Oskar Laverny and Santiago Jimenez. "Copulas.jl: A fully Distributions.jl-compliant copula package". In: *Journal of Open Source Software* 9.94 (2024), p. 6189. DOI: [10.21105/joss.06189](https://doi.org/10.21105/joss.06189). URL: <https://doi.org/10.21105/joss.06189>.

²⁰⁴Robin A.A. Ince et al. "A statistical framework for neuroimaging data analysis based on mutual information estimated via a gaussian copula". In: *Human Brain Mapping* 38.3 (2017), pp. 1541–1573, Robin A.A. Ince et al. *gcmi : Gaussian-Copula Mutual Information*. GitHub. URL: <https://github.com/robinince/gcml>. 2020.

²⁰⁵Robert Oostenveld et al. "FieldTrip: Open Source Software for Advanced Analysis of MEG, EEG, and Invasive Electrophysiological Data". In: *Computational Intelligence and Neuroscience* 2011.1 (2011), p. 156869. DOI: <https://doi.org/10.1155/2011/156869>.

²⁰⁶Igor Volobouev. *NPStat – Non-parametric statistical modeling and analysis in C++ and Python*. HEPForge. URL: <https://npstat.hepforge.org>. 2023.

²⁰⁷The CE estimation method is decomposed into two functions for empirical copula estimation and entropy estimation.

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